

# NEW NON-UNIFORM WAVEGUIDE TAPER DESIGN YIELDING LOW VSWR AND HIGH REJECTION

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## Abstract

A new and improved theoretical method is described for the design of continuous exponential waveguide tapers which may be utilized in designing high pass filters or transitions between different sized waveguides. Comparisons are made between the new approach and commonly published designs. Experimental results from a filter, with a 1% separation between the passband and the stop band, designed by the new method are presented.

## Summary

## Introduction

Waveguide high pass filters are commonly designed by reducing the internal dimensions of a section of the waveguide such that all frequencies at or below a waveguide cutoff frequency are rejected. This may be done in rectangular waveguide by decreasing the width, commonly referred to as the "a" dimension, or by decreasing the diameter in circular waveguide. The amount of rejection of the filter can be determined by controlling the length of the cutoff waveguide section, while the reflected loss in the passband will be determined by the design of the transition between the two waveguide sizes. Often the filter will be required to pass frequencies which are critically close to the cutoff frequency,  $f_c$ . In this case great care must be given to the design of a well matched transition because the waveguide impedance and the propagation factor will undergo large changes near  $f_c$ .

The design approach discussed here is also recommended for use in transitioning between different waveguide sizes where filtering is not the main objective.

## Theoretical Aspect

The input reflection coefficient,  $\Gamma$ , of a non-uniform dispersive transmission line (with a matched load) is given by

$$\Gamma = \frac{1}{2} \int_0^\Theta e^{-j2\theta} \frac{d(\ln Z_c)}{d\theta} d\theta, \text{ for } |\Gamma| \ll 1 \quad (1)$$

where

$Z_c$  = Characteristic Impedance of Waveguide

$$\theta = \int_0^x \beta(x) dx$$

$$\theta_0 = \int_0^L \beta(x) dx, \text{ electrical length of the transmission line}$$

For a TEM line  $\beta$  is constant and equation (1) reduces to

$$\Gamma = \frac{1}{2} \int_0^L e^{-j2\beta x} \frac{d(\ln Z_c)}{dx} dx \quad (2)$$

In order to design a well-matched continuous taper with a varying propagation factor one should use an expression which distributes the impedance as a function of electrical length,  $\theta$ . However, if  $Z_c$  is expressed in terms of  $\theta$ , it may be rather difficult to translate it into a function of  $x$ . Thus, the expression commonly recommended in the literature which only distributes the impedance as a function of physical length is shown in equation (3).<sup>2,3,4</sup> (See Figure 1 for coordinates.) This expression gives good results for TEM type of transmission lines where  $\beta(x)$  is a constant, but it is not ideally suited for waveguides where  $\beta(x)$  does vary.

The expression proposed by this paper that distributes the impedance of a lossless line as a function of electrical length,  $\theta$ , is shown in equation (4). Using this expression provides a natural exponential distribution of impedance which will provide a good match at the design frequency,  $f_0$ .

$$Z_c(x) = \sqrt{Z_1 Z_2} e^{R \cdot \cos(\frac{\pi x}{L})} \quad (3)$$

$$Z_c(\theta) = \sqrt{Z_1 Z_2} e^{R \cdot \cos(\frac{\pi \theta}{L \beta})} \quad (4)$$

where,

$$R = \ell \ln \sqrt{\frac{Z_1}{Z_2}} \text{ and } \beta = \frac{1}{L} \int_0^L \beta(x) dx$$

A third expression shown in equation (5), which will be used for comparison, is for a conventional exponential line.

$$Z_c(x) = Z_1 e^{\frac{x}{L} \ell \ln (Z_2/Z_1)} \quad (5)$$

A filter of 12 inches total length consisting of two 6-inch H-Plane tapers was designed from each of equations (3), (4), and (5). The design from equation (4) was done by the numerical technique discussed later in this paper. The responses of each of the filters were computed by using numerical methods. They were designed from WR137 rectangular waveguide with the following parameters.

$$f_0 = 7.9 \text{ GHz (Design frequency)}$$

$$f_c = 7.827 \text{ GHz (Cutoff frequency)}$$

$$a_1 = 1.372 \text{ IN}$$

$$a_2 = .754 \text{ IN}$$

$$b = .622 \text{ IN (Constant)}$$

$$L = 6 \text{ IN}$$

The profiles of the tapers of these filters are shown in Figure 2. Note that the taper designed from equation (4) tapers very rapidly from the large width,  $a_1$ , and very slowly to the narrow width,  $a_2$ . This results from distributing the impedance,  $Z$ , as a function of  $\theta$ .

The responses of the three designs are shown in Figure 3. Equation (4) not only gives the lowest VSWR at the design frequency,  $f_0$ , but it is also lower in VSWR response for the rest of the passband. Equation (4) also provides more rejection at frequencies below  $f_0$  because of its more rapid taper. It must be borne in mind that a continuous taper presents reactances which will distort the mode structure of the propagating fields. This effect, which will cause slight deviations from the computed behavior, can be minimized by lengthening the taper.

#### Numerical Design Technique

A technique to design a waveguide taper of a predetermined length,  $L$ , using equation (4) is to first proportion the impedance steps logarithmically into  $N$  steps by using equation (6).

$$z_c(\theta(x_i)) = z_1 e^{\frac{i}{N} \ln(z_2/z_1)}, \quad i = 1, 2, \dots, N \quad (6)$$

where  $x_i$  is the position where (6) is satisfied. From equation (4), we obtain

$$\frac{\theta(x_i)}{\beta} = \frac{L}{\pi} \cos^{-1} \left( 1 - \frac{2i}{N} \right) \quad (7)$$

where,

$$\theta(x_i) = \int_0^{x_i} \beta(x) dx$$

Our last step is to find  $x_i$ , step position, after  $\theta(x_i)$  is obtained from (7). For large  $N$  this can be done by using the following derived numerical expressions.

$$x_i = \sum_1^i \Delta x_i, \quad i = 1, 2, \dots, N$$

where,

$$\Delta x_i = \bar{\beta} \cdot \left( \frac{\Delta \bar{\theta}_i}{\beta(x_i)} \right)$$

$$\bar{\beta} = \frac{L}{\sum_{i=1}^N \frac{\Delta \bar{\theta}_i}{\beta(x_i)}}$$

$$\Delta \bar{\theta}_i = \left( \frac{\theta(x_i)}{\bar{\beta}} - \frac{\theta(x_{i-1})}{\bar{\beta}} \right)$$

Note that  $\beta(x_i)$  can be obtained from  $z_c(\theta(x_i))$ . This technique can be implemented with the aid of a digital computer.

The partition number ( $N$ ) affects the performance of the filter, as can be seen from the various curves in Figure 4., where comparisons have been made for  $N = 10, 20, 30$  and  $40$ . The larger the value of  $N$ , the closer the stepped profile will approach the original smooth profile of the taper. In this example the reflection characteristics show little change for  $N \geq 30$ .

#### Experimental Result

A high pass filter was built in circular waveguide (shown in Figure 5) using equation (4) as the design. This filter tapers from a diameter of 0.720 inches to a diameter of 0.583 inches with a cutoff frequency of 11.866 GHz and a design frequency of 12.0 GHz. The impedance transforms from an impedance of 1258 ohms to 5070 ohms (ratio 4:1) in an electrical length of 262 degrees at 12.0 GHz. It has a total length of 5.75 inches consisting of 2.75 inches for each taper and a cutoff section of 0.25 inches. Good correlation is shown between the computed and measured responses as shown in Figure 6. A machining error of .001 inch in the small diameter caused a slight shift of the frequency response.

#### Conclusion

A new theoretical method for designing continuous waveguide tapers and high pass filters has been presented that is useful for obtaining low reflections near the cutoff frequency. The design is most useful when both high rejection values and low VSWR's are required and mechanical constraints limit the length of the filter. Since the design is simple to use, it is also recommended for transitioning between two different sized waveguides where filtering is not the objective.

#### References

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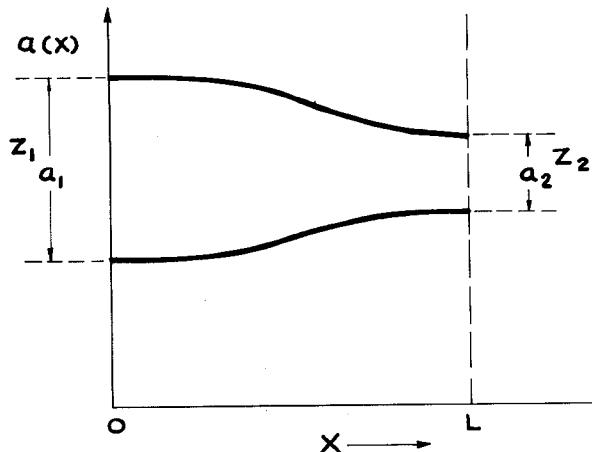


FIGURE 1  
A WAVEGUIDE TAPER SECTION

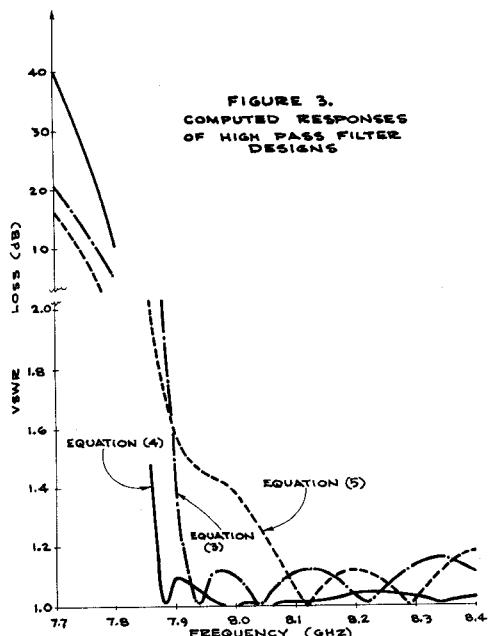


FIGURE 3.  
COMPUTED RESPONSES  
OF HIGH PASS FILTER  
DESIGNS

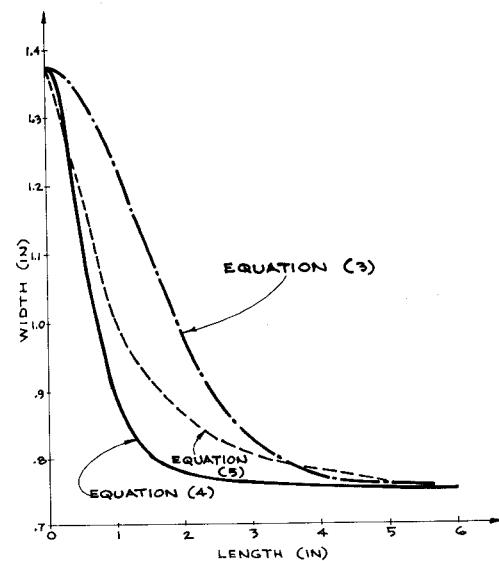


FIGURE 2. TAPER PROFILES

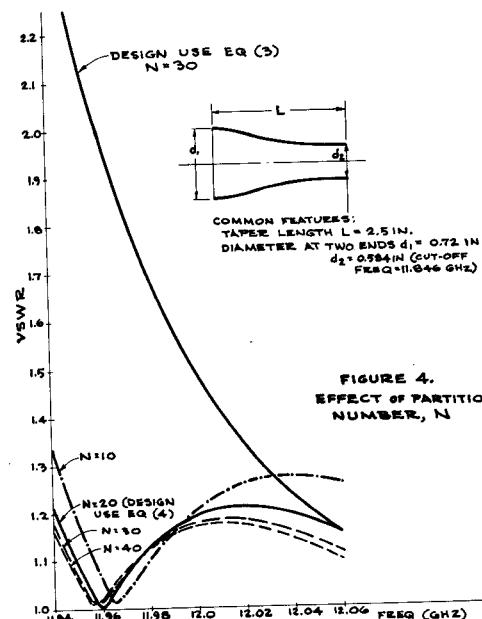


FIGURE 4.  
EFFECT OF PARTITION  
NUMBER, N

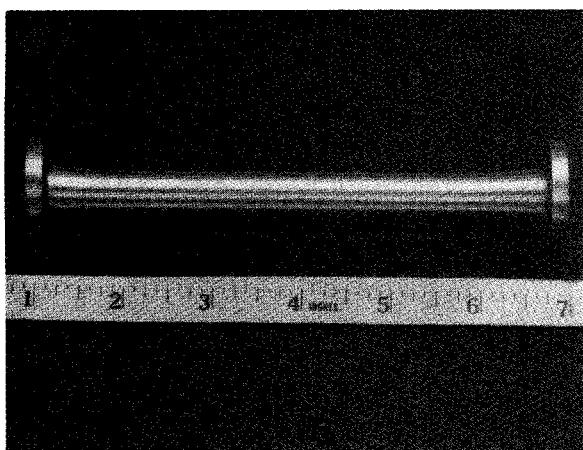


FIGURE 5  
EXPERIMENTAL MODEL OF 12 GHz  
HIGH-PASS FILTER

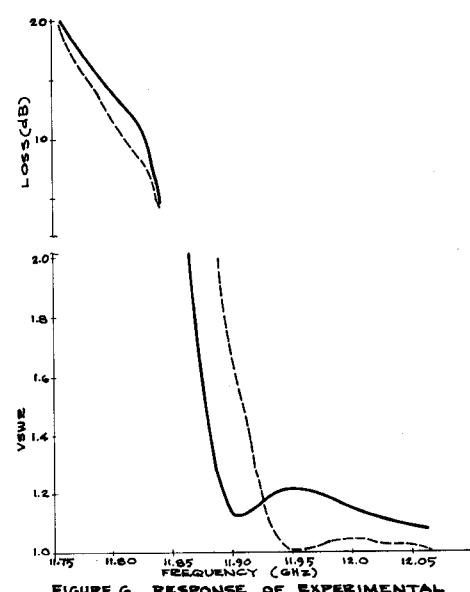


FIGURE 6. RESPONSE OF EXPERIMENTAL  
MODEL

NOTES